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MATHEMATICAL MODELING OF EVAPORATIVE COOLING OF WATER IN A MECHANICAL-DRAFT TOWER

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A mathematical model of the operation of a mechanical-draft tower is proposed. The model represents a boundary-value problem for five differential equations and for the first time takes into account the following parameters: temperature of inflowing water, its discharge, mean radius of water droplets, mean air velocity inside the water-cooling tower, rate of fall of a droplet, height from which droplets fall, and meteorological conditions, including atmospheric pressure. The basic parameters of the water-cooling tower influencing its thermal efficiency have been revealed. The dissipation of the kinetic energy of the air flow due to the air friction against the surface of falling droplets has been taken into account.

Introduction. In industry, for stable and deep cooling of circulating water, especially at high specific water concentrations and under high unit thermal loads, counterflow mechanical-draft drip-type cooling towers are widely used [1–3]. They incorporate the following basic elements: frame, water distributor, water catcher, catchment basin, and induced-draught fan.

In the present paper, we restrict ourselves to the consideration of mechanical-draft towers in which only a drip-type flow of the liquid phase takes place. As a rule, in water atomizing, the spray nozzles form a polydisperse ensemble of droplets. Note that a pilot model of a cooling tower with monodisperse droplets has already been designed [4].

For mathematical modeling of the evaporative cooling of water in a mechanical-draft tower, the approach previously developed to calculate the parameters of a chimney-type cooling tower is used [5]. In so doing, to simplify the mathematical model, we assume that all falling droplets in the cooling tower have the same radius. The evaporative cooling of water is calculated as the solution of the boundary-value problem for two thermal agents moving in opposite directions. As droplets are falling, evaporation of water and a convective heat exchange with cooler air vapors occur. With increasing rate of fall of droplets, the time of interaction with fresh, relatively cold air decreases. On the other hand, as the air rises, its heating and saturation with water vapors occurs, which retards the rate of heat-and-mass exchange in the process of water cooling.

We describe the internal aerodynamics of the mechanical-draft tower in the one-dimensional approximation by the mean air-flow rate u, constant with respect to height and cross section, so that in the air flow only the temperature and humidity change. Unlike cooling towers with natural draft, where the convection rate depends on the degree of air cooling and its saturation with water vapors, in the case under consideration the air-flow rate u is determined by the fan power and the total aerodynamic drag. The heat-and-mass exchange processes in the cooling tower depend on the specific mass flows of water Q_w and air Q_a, the temperature T_{a0} and humidity of the air flowing into the cooling tower ψ, the temperature of the inflowing water T_{w0}, the wind velocity, and the atmospheric pressure [6].

We shall characterize the evaporative cooling efficiency by the dimensionless parameter η [7]:

$$\eta = \frac{T_{w0} - T_{wf}}{T_{w0} - T_{lim}} . \tag{1}$$

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The limiting water-cooling temperature at evaporative cooling T_{lim} has been calculated for the air temperature T_{a0} and its relative humidity ψ. The quantity T_{lim} is equal to the wet-bulb thermometer temperature and is determined from the equation

$$\rho_s (T_a) \psi = \rho_s (T_{lim}) . \tag{2}$$

From the above qualitative physical description as well as from the notions of the dimensional theory [8], the formula

$$\eta = F (Q_w / Q_a, H / R, (T_{a0} - T_{w0}) / (T_{w0} - T_{lim})) , \tag{3}$$

where F is the function of the dimensionless arguments, follows.

The mathematical model of evaporative cooling presented below permits finding the characteristic features of the function F. On the other hand, the right side of (3), obtained on the basis of the general ideas of the dimensional theory, makes it possible to considerably reduce the volume of the computing experiment.

Mathematical Model. Droplets in the mechanical-draft tower are formed by water atomization by spray nozzles (atomizers). The droplet radius depends on the water discharge in the cooling tower: the larger the water discharge, the smaller the size of the droplets because of the high differential pressure on the atomizers. As the calculations show, the dependence of the droplet radius on the water concentration is due to the design features of the nozzle and is not associated with the droplet breakdown. Even at a maximum water concentration the rate of fall of droplets at the nozzle exit is not sufficient for breaking them.

The maximum and minimum values of the droplet radii in the counterflow cooling tower are determined, respectively, by the breaking of large drops and the carrying-away of small ones by the air flow. The maximum radius of a droplet falling at a velocity v is found from the condition of equality of the aerodynamic drag forces and the surface tension. Droplets of radius R do not break if the equality [9]

$$R \le 2.3 \frac{\sigma}{\rho_a v^2} . \tag{4}$$

is fulfilled. The minimum size of droplets participating in the process of evaporative cooling depends on the rising airflow rate u. If the aerodynamic drag force is greater than the force of gravity, which is true for sufficiently small droplets, then they are carried away by the rising air flow:

u, m/sec	0.5	1	1.5	2	2.5
$R_{min} \cdot 10^{-4}$, m	0.5	1.0	1.5	2.0	2.5

We now turn to the calculation of the evaporative cooling of water droplets. We direct the z axis vertically downward and bring the origin of coordinates into coincidence with the starting point of the droplet fall. The droplet in free fall is subjected to the action of gravity and the aerodynamic drag force. As the vapor-air flow rises, its temperature and humidity increase and the droplet temperature decreases. The system of differential equations for calculating the characteristics of the processes of heat-and-mass exchange of the accelerating droplet in the ascending air flow includes the following:

the equation describing the change in the radius $R(z)$ of the droplet due to its evaporation:

$$\frac{dR (z)}{dz} = - \frac{\gamma (\mathrm{Re}) [\rho_s (T_w (z)) - \rho (z)]}{\rho_w v (z)} ; \tag{5}$$

the equation describing the change in the rate of fall of the droplet $v(z)$ (note that for relatively low mechanical-draft towers the account for the accelerated motion of droplets is of fundamental importance, since the rate of fall of the droplet does not permit it to reach a steady rate of fall):

$$\frac{dv (z)}{dz} = \frac{g}{v (z)} - C (\mathrm{Re}) \frac{\rho_a [v (z) - u]^2}{2v (z)} \frac{\pi R (z)^2}{m} . \tag{6}$$

Equation (6) can practically be solved at a constant value of the droplet radius, since it changes at evaporation by less than 1%;

the equation describing the change in the volume-mean temperature of the droplet $T_w(z)$:

$$\frac{dT_w(z)}{dz} = \frac{3\left\{\alpha(\mathrm{Re})\,[T_a(z) - T_w(z)] + \gamma(\mathrm{Re})\,(r - c_w T_w(z))\,[\rho_s(T_w(z)) - \rho(z)]\right\}}{c_w \rho_w R(z)\, v(z)}, \tag{7}$$

and it can easily be shown that for fine droplets, of the order of 1 mm, the difference between the surface temperature and the volume-mean temperature is very small;

the equation for calculating the change in the vapor-air mixture temperature $T_a(z)$ (note that the rate of change in the air temperature is directly proportional to the interface area $4\pi R^2 N$ and inversely proportional to the relative phase velocity):

$$\frac{dT_a(z)}{dz} = \frac{4\pi R(z)^2 N(z)}{\rho_a c_a (v(z) - u)}\,[\alpha(\mathrm{Re})\,[T_a(z) - T_w(z)]]; \tag{8}$$

and the equation for describing the change in the water-vapor density $p(z)$ in air:

$$\frac{d\rho(z)}{dz} = -\frac{4\pi R(z)^2 N(z)}{v(z) - u}\,\gamma(\mathrm{Re})\,[\rho_s(T_w(z)) - \rho(z)]. \tag{9}$$

The boundary conditions for the system of equations (5)–(9) for droplets are written as follows:

at $z = 0$ (the starting point of the droplet fall)

the droplet radius

$$R\big|_{z=0} = R_0, \tag{10}$$

the droplet temperature

$$T_w\big|_{z=0} = T_{w0}, \tag{11}$$

the rate of fall of the droplet

$$v\big|_{z=0} = 0; \tag{12}$$

at $z = H$

the air temperature

$$T_a\big|_{z=H} = T_{a0}, \tag{13}$$

the water-vapor density in air

$$\rho\big|_{z=H} = \rho_0. \tag{14}$$

Thus, the system of ordinary differential equations (5)–(9) and the boundary conditions (10)–(14) represent a nonlinear boundary-value problem.

Note that our model has taken into account the influence of the number of droplets on the parameters of the vapor-air medium. The number of droplets in a unit volume $N(z)$ is determined as

$$N(z) = \frac{3Q_w}{4\rho_w \pi R^3 v(z)}. \tag{15}$$

As follows from (15), for constant water discharge with increasing rate of fall of droplets their number in a unit volume decreases. As a rule, for mechanical-draft towers [2, 3] the water concentrations are such that the mean distance between droplets is much larger than their radius. This fact has been taken into account in our model.

The coefficient of heat exchange of the droplet with the air medium α that enters into (7) and (8) is determined, according to [10], from the following criterion dependence:

$$\mathrm{Nu} = 2 + 0.5\,\mathrm{Re}^{0.5}\,. \tag{16}$$

For droplets the Nusselt number $\mathrm{Nu} = 2R\alpha(\mathrm{Re})/\lambda_a$ and the Reynolds number:

$$\mathrm{Re} = \frac{2\rho_a R\,[(v-u)^2]^{0.5}}{\mu_a}\,. \tag{17}$$

The coefficient of mass exchange of the droplet in free fall with ascending air γ is determined from the analogy between the processes of heat- and mass exchange as

$$\gamma = \frac{D\,(2 + 0.5\,\mathrm{Re}^{0.5})}{2R\,(z)}\,. \tag{18}$$

The aerodynamic drag coefficient of the droplet $C(\mathrm{Re})$ is calculated by the formula [9]

$$C\,(\mathrm{Re}) = \frac{24}{\mathrm{Re}}\left(1 + \frac{1}{6}\,\mathrm{Re}^{2/3}\right). \tag{19}$$

In the numerical simulation, the temperature dependence of the diffusion constant and the coefficient of viscosity are taken into account. Before proceeding to the numerical investigation of the system of equations (5)–(14), we give qualitative estimates of the evaporative cooling of droplets obtained by the approximate analytical integration of the above system of equations [11]. For the change in the droplet temperature ΔT_w in the mechanical-draft tower, where accelerated motion of water occurs, we have the qualitative estimate

$$\Delta T_w \sim \left\{\lambda\,[T_{a0} - T_{w0}] + Dr\,[\rho_s\,(T_{w0}) - \rho_0]\right\} H^{0.5} u^{0.5} R_0^{-3/2}\,. \tag{20}$$

Interestingly, the estimate of ΔT_w is directly proportional to $(Hu)^{0.5}$, which is in good agreement with the data of numerical calculations given below. Note the inverse relationship between ΔT_w and the initial radius of the droplet R_0. Transformation of (20) yields

$$\eta \sim \left\{\lambda\,\frac{[T_{a0} - T_{w0}]}{(T_{w0} - T_{\lim})\,R_0} + Dr\,\frac{[\rho_s\,(T_{w0}) - \rho_0]}{(T_{w0} - T_{\lim})\,R_0}\right\}\left(\frac{H}{R_0}\right)^{0.5} u^{0.5}\,. \tag{21}$$

From expression (21) it is seen that for the mechanical-draft tower the thermal efficiency depends on many parameters, and the contribution of the second term thereby is dominating. Since the diffusion constant $D \sim 1/P$, the thermal efficiency of the cooling tower depends on the atmospheric pressure as well.

The solution of the boundary-value problem of evaporative cooling for droplets (5)–(14) was carried out by the "shoot" method [12]. For the numerical solution of the differential equations, we used the fourth-order Runge–Kutta scheme. The accuracy of the calculations was controlled by means of the residual criterion E:

$$\Sigma\,(T_{a0}, \rho_0) = \sqrt{\left(\frac{T_a\,(H) - T_{a0}}{T_{a0}}\right)^2 + \left(\frac{\rho\,(H) - \rho_0}{\rho_0}\right)^2}\,, \tag{22}$$

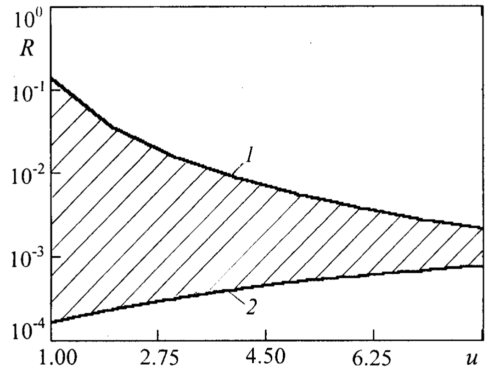

Fig. 1. Possible values of droplet radii (shaded area) versus the ascending airflow rate: 1) for the largest possible radius of droplets; 2) for the least possible radius. R, m; u, m/sec.

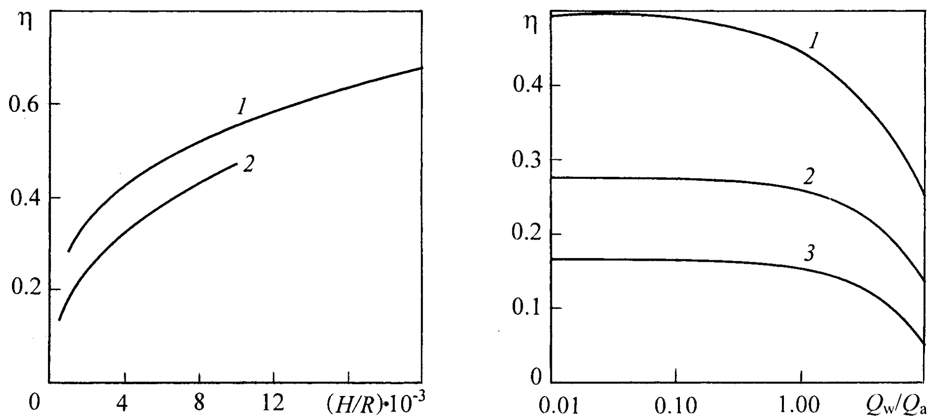

Fig. 2. Thermal efficiency η versus the dimensionless height from which droplets fall H/R (a) and Q_w/Q_a (b): (T_0 = 40°C, T_{a0} = 20°C, ψ = 60%, Q_w = 1.74 kg/(m^2·sec), u = 2 m/sec): 1) R = 0.5 mm, 2) 1; 3) 1.5.

where $T_a(H)$ and $\rho(H)$ are the results of the solution of (5)–(14) for the air temperature at the point of its inflow. The solution of the problem terminates when the condition $\Sigma < 10^{-4}$ is met.

Comparison of the results of the calculations by the proposed mathematical model (5)–(14) with the available experimental data [13] has shown that our model provides a qualitatively correct description of the cooling of the water droplet in free fall in air. In this case, the ratio of the calculated value of the droplet temperature drop ΔT_{theor} to the experimental one ΔT_{exper} for different heights H does not exceed 10%.

Results of Calculations. The results of the calculations by the mathematical model (5)–(9) with boundary conditions (10)–(14) are presented in Figs. 2 and 3.

Figure 1 shows the dependence of the ultimate radii of water droplets in free fall in the mechanical-draft tower on the ascending air velocity. Curve 1 corresponds to the largest possible radius of droplets found in accordance with expression (3) and curve 2 — to the least possible radius because of the carrying away.

As seen from Fig. 1, in the mechanical-draft tower, depending on the ascending air velocity, droplets with values of the radii between curves 1 and 2 can be present. If the droplet radius falls into the region above curve 1, then such droplets are broken by the air flow, and if below curve 2 — they are carried away by the ascending air. Note that the droplet radius equal to 1 mm is the most probable value at a water temperature of 20°C. All other things being equal, an increase in the water temperature decreases the radius value because of the decrease in the surface tension coefficient.

As follows from the calculated data presented in Fig. 2a, the dependence of the thermal efficiency of the water-cooling tower on the distance of fall of the droplet is nonlinear, which qualitatively agrees with expression (21). One can clearly see the saturation effect resulting from the increase in the vapor-air mixture humidity and temperature.

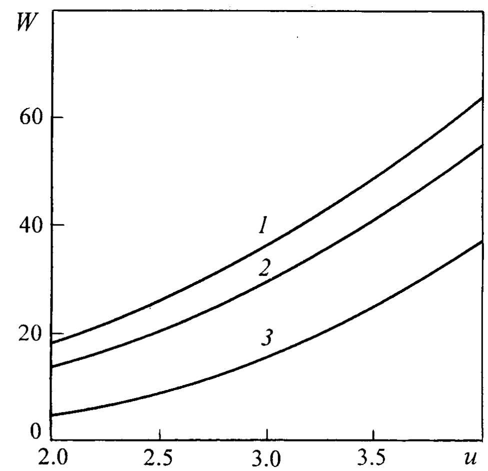

Fig. 3. Specific power loss of the flow W versus its rate u: 1) Q_w = 2.6 kg/(m^2·sec); 2) 1.7; 3) without "rain drag" (H = 2 m, R_0 = 1 mm). W, (J·m)/sec; u, m/sec.

An important role is also played by the increase in the rate of fall of droplets with increasing height H so that the time of interaction of droplets with cold air decreases.

From Fig. 2b it is seen that the thermal efficiency of the water-cooling tower η decreases with increasing ratio Q_w/Q_a. This dependence is practically absent for small values of Q_w/Q_a, which corresponds to either a small water concentration or a large air flow through the cooling tower. In this case, the evaporative cooling efficiency is determined in the initial part of the droplet flight, since the thermodynamic constraints are absent. It is seen that the economically optimum mode of operation of the mechanical-draft tower corresponds to $Q_w/Q_a \approx 1$. The dependence of the thermal efficiency of the cooling tower η on the droplet radius is rather strong and is in qualitative agreement with formulas (20) and (21).

Let us estimate the kinetic energy dissipation of the air flow in the mechanical-draft tower due to the friction of droplets against the rising flow. The power density W lost by the air flow due to friction is directly proportional to the mass flow of water and, naturally, depends on the droplet radius and height

$$W = Q_w \frac{(v_1 (R, H)^2 - v_2 (R, H)^2)}{2}, \tag{23}$$

where $v_1(R, H)$ and $v_2(R, H)$ are the velocities of water droplets of radius R in their free fall from height H in air at rest and in air ascending at the rate u respectively. Figure 3 shows the dependence of the specific power density W on the air-flow rate for the tower height H = 2 m, the initial droplet radius R = 1 mm, and two values of the specific mass flow of water, Q_w = 1.7 and 2.6 kg/(m^2·sec). It is seen that the "rain drag" in the cooling tower requires a significant increase in the fan power to operate in a given aerodynamic regime, and the higher the tower, the greater the contribution of this drag.

The proposed mathematical model of droplet cooling makes it possible to calculate the vaporized water discharge in the steady-state regime of operation of the mechanical-draft tower. As the calculations have shown, it increases from 1.5 to 3% as the air-flow rate increases from 2 to 4 m/sec (at fixed R = 0.001 m, T_0 = 40.7^{o}C, T_a = 23.4^{o}C, ψ = 0.36, and Q_w = 2.6 kg/(m^2·sec)). In so doing, the thermal efficiency of the cooling tower increases from 25 to 33%.

CONCLUSIONS

For the drip-type counterflow mechanical-draft tower, a one-dimensional mathematical model representing a boundary-value problem for a system of nonlinear ordinary differential equations describing the interrelated heat-and-mass exchange processes and the fall dynamics of droplets has been developed. This model assumes that all droplets in the cooling tower have the same radius *R*. The qualitative estimates for the temperature drop of water droplets and the thermal efficiency of the drip-type mechanical-draft tower are given.

It has been revealed that the thermal efficiency of the cooling tower η strongly depends on the ratio H/R. Depending on the droplet radius, all other things being equal, there exists a limiting height of the mechanical-draft tower at which the maximum efficiency is attained.

The dependence of the thermal efficiency of the drip-type mechanical-draft tower on the ratio between the mass flows of water and air has been determined. When this ratio changes from 0.01 to 1, the thermal efficiency of the cooling tower remains practically unchanged. From the economic point of view the most advantageous mode of operation of the mechanical-draft tower corresponds to $Q_w/Q_a \approx 1$.

It has been shown that the mean radius of droplets strongly influences the efficiency of the cooling tower operation. It seems to be expedient to further develop the above-described mathematical model, taking into account the radius distribution of droplets.

The vaporized water discharge in the steady-state regime of operation of the cooling tower has been calculated. It can reach 3% of the mass flow of water, and this value thereby depends on the atmospheric pressure.

It has been shown that the compensation for the kinetic energy dissipation of the air flow due to the "rain drag" requires a considerable increase in the fan power. The higher the cooling tower, the greater the contribution of this drag. Accounting for the two-dimensional aerodynamic effects inside the mechanical-draft tower, especially at the entrance, will make it possible to increase the predictability of the model. Work in this direction is under way.

NOTATION

c, heat capacity, J/(kg·K); *D*, diffusion constant of water vapors into air; *R*, water-droplet radius, m; *H*, height from which droplets fall, m; *m*, droplet mass, kg; *r*, latent heat of vaporization, J/kg; *u*, air-flow rate, m/sec; *v*, droplet velocity, m/sec; *P*, atmospheric pressure; *Q*, specific mass flow, kg/(m^2·sec); *T*, temperature, ^{o}C; *g*, free-fall acceleration, m/sec; *N*, number of droplets; *C*, aerodynamic drag coefficient; *W*, power density, (J·m)/sec; α, heat-exchange coefficient; γ, mass-exchange coefficient; λ, heat conductivity of air, W/(m·K); μ, dynamic viscosity of air, kg/(m·sec); ψ, relative humidity of air; η, thermal efficiency of the cooling tower; ρ, density, kg/m^3; σ, surface tension coefficient of water; Re, Reynolds number; Nu, Nusselt number. Subscripts: 0, initial value (at the entrance to the cooling tower); f, final value (at the exit from the cooling tower); s, saturated water vapors; a, air; w, water, lim, limiting value; theor, theoretical; exper, experimental; min, minimum.

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